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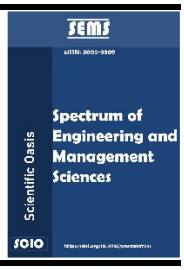

Decision-Making for the Bakery Product Transportation using Linear Programming

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ABSTRACT

This research paper presents a case study of a food chain in Pakistan that aims to transport its bakery products from the manufacturing unit in Islamabad to remote northern regions of the country while minimizing the total cost of distribution and transportation. The study employs various analytical methods, including linear programming, network optimization, and simulation modelling, to identify the optimal transportation solution that considers geographical barriers, infrastructural limitations, seasonal fluctuations, and regulatory constraints. The research delves into the underlying factors that contribute to the complexity and challenges of delivering goods to remote areas and explores the potential benefits of adopting a more holistic and integrated approach to transportation management. The study provides valuable insights and actionable recommendations for the Pakistani food chain and contributes to the broader field of transportation and logistics management.

1. Introduction

The rapid growth of global trade and the increasing complexity of international supply chains have highlighted the importance of efficient transportation and logistics management in today's business environment. In this context, the ability of manufacturing companies to effectively transport their products to customers, particularly in remote or underdeveloped regions, has emerged as a critical determinant of their competitiveness and overall success [1,2]. Such logistical challenges can be particularly daunting for companies that lack the necessary resources or expertise to manage their transportation operations in-house, necessitating the involvement of specialized third-party logistics (3PL) providers [3].

This research paper offers an extensive case study of a prominent Pakistani food chain brand that encounters complex transportation challenges in delivering its bakery items to customers in remote areas. The primary objective of this case study is to identify the most effective and efficient

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transportation strategies for the company's products, considering the logistical constraints and challenges associated with operating in these regions [4]. To address this problem, we employ a comprehensive suite of operational research techniques, including linear programming, network optimization, simulation modeling, multi-criteria decision analysis, and heuristic algorithms. These methodologies enable us to systematically evaluate and compare various transportation strategies, considering factors such as cost, delivery time, reliability, environmental impact, and the company's overall supply chain performance [5,6].

Therefore, if a company wants to move its five different products to five different places at five different times, then it can create a complex problem for the manufacturing company. Every manufacturing company would want to minimize the cost of the production unit to save money and increase profit. Transportation companies have different ways to move goods from source to destination. In our analysis, we delve into several alternative transportation solutions, such as the use of different transport modes (e.g., road, rail, air, and maritime), consolidation of shipments, strategic location of distribution centers, implementation of advanced technologies to optimize routing and scheduling, and collaboration with local partners to leverage their knowledge and expertise in operating in remote regions [7,8]. Furthermore, we explore the potential benefits of adopting a holistic and integrated approach to transportation management, incorporating elements of risk management, sustainability, and resilience into the decision-making process [9,10]. This approach recognizes the interdependencies and trade-offs between various transportation decisions, as well as their implications for the company's broader supply chain performance and long-term competitiveness [11]. Through our exhaustive investigation, we aim to provide valuable insights and actionable recommendations to the Pakistani food chain brand, as well as other manufacturing companies facing similar transportation challenges in remote areas. Additionally, this research contributes to the wider body of knowledge in the field of transportation and logistics management, emphasizing the significance of operational research methodologies in tackling complex, real-world problems and informing decision-making processes [12].

The main idea of this study is to find out the best way to transport the bakery items of a famous Pakistani food chain brand in remote areas, where transportation is not easy. Moreover, our research underscores the need for collaborative partnerships between manufacturing companies, transportation service providers, and other supply chain stakeholders to develop innovative and sustainable transportation solutions that cater to the unique logistical requirements of remote regions [13,14]. Such collaborative efforts can facilitate the development of more efficient, resilient, and environmentally friendly supply chains, which not only enhance the competitiveness of individual companies but also contribute to the socioeconomic development and well-being of remote communities [15].

This paper is structured as follows: Section 2 overviews the related literature, while the next section presents the methodology as well as methods to find primary feasible solutions. Section 4 gives results, while the last section provides the conclusions.

2. Literature Review

Ajibade & Babarinde [16] introduced the concept of using transportation techniques to identify the most cost-effective means of shipping commodities from various supply sources to different demand destinations. This innovative approach highlighted a practical method for determining the lowest shipping costs for goods between multiple locations. Various researchers have investigated different methods and models to solve transportation problems to further explore and refine this idea. Shipping operations are impacted by disruptions, which can decrease schedule reliability and potentially raise the overall cost of delivering cargo at the required destination points [16,17]. In

destination points, the frequent mentions include the capacity mismatch between the points of destination and logistics tasks, shortages of labor and equipment, as well as weather events [18].

Gumuskaya *et al.* [19] emphasize the significance of coordination and conflicting incentives in logistics operations, particularly concerning the emergence of congestion. Loch & Silva [20] examined the concept of degeneracy in initial simple feasible solutions for transportation issues. They discovered that the preliminary simple solutions obtained were not necessarily fundamental and required comparison with degenerated variables. They employed the least-cost technique to determine transportation problem solutions at different levels to investigate the presence of degenerated variables. They proposed a mathematical model that could be applied iteratively to find transportation problem solutions. Their research demonstrated that the least-cost method and Vogel's approximation method were the most appropriate techniques for solving larger transportation issues. This finding significantly contributed to the optimization of transportation cost calculations in various scenarios. Kumaraguru *et al.* [21] and Verma & Vanita [22] conducted a comparative study of different methods for solving transportation problems. Their research ultimately determined that the zero-suffix method achieved optimal results in their numerical example. However, they acknowledged that no single method could be universally considered the best or most optimal solution for all transportation problem scenarios. Afroz & Hassan [23] and Bera & Mondal [24] explored a computer-based technique for addressing transportation issues. They found that manual calculations using existing methods became increasingly time-consuming as the number of variables increased. Consequently, implementing mathematical software to automate calculations proved to be a significant time-saving measure. This discovery emphasized the importance of incorporating technology in solving complex transportation problems, streamlining the process, and ultimately reducing overall costs.

Overall, the reviewed literature illustrated the ongoing development and refinement of methods and models for solving transportation problems. These studies have contributed valuable insights into identifying cost-effective shipping solutions and have highlighted the potential for technological advancements to further optimize the transportation industry.

3. Methodology

3.1 Transportation and Distribution Problem

Transportation and distribution problem helps in decision-making processes within the realm of managing extensive enterprises or structured systems [25,26]. The biggest challenge in production distribution and transportation is to find out a schedule that can minimize the transporting cost of the products from the source points to different destination points by several suppliers. The model must be in a balanced form before we start working on the solution. It means that the sum of supply must be equal to the total demand. If it is not balanced, then it will not be possible to find out the best optimal solution for this issue. An extra dummy row or column can be added in the table to balance the data.

3.2 Mathematical Statement of Transportation and Distribution Problem

The transportation issue falls within the category of linear programming problems and can be effectively addressed by employing the simplex method. It finds significant application in resolving challenges about multiple sources of products and diverse destinations for goods. The problem statement provides the necessary information about the quantity demanded by each destination, which serves as a crucial factor in determining the optimal transportation plan. In this problem, we are given the cost of shipping a single unit of goods from a non-origin location (i.e., a plant or warehouse) to a non-destination point (such as a distribution center or another intermediate

location). This information is critical to the formulation of the transportation model, as it helps in understanding the underlying cost structure and the potential trade-offs in the transportation process. It is important to note that the non-origin and non-destination locations are intermediate points in the transportation process, which may further influence the total shipping cost. The primary objective of this research is to determine the minimum total shipping cost that can be achieved while adhering to the given constraints and meeting the demand of each destination. This can be accomplished by employing linear programming techniques, which involve the formulation of a mathematical model that represents the problem scenario. The model will consist of decision variables, an objective function, and a set of constraints. The decision variables in this case will represent the quantity of goods to be transported from each plant to each destination or intermediate location.

The objective function will aim to minimize the total cost of shipping, which is computed as the sum of the individual shipping costs from each non-origin to each non-destination location, multiplied by the corresponding decision variables. The constraints will ensure that the quantity supplied by each plant, the quantity demanded by each destination, and the capacity of each intermediate location are all considered. By solving this linear programming problem, we will be able to identify the optimal transportation plan that minimizes the total shipping cost while meeting the demands of all destinations. This research will contribute to the growing body of knowledge on transportation optimization and has the potential to inform the development of more efficient and cost-effective transportation strategies in various industries.

The objective function is given by:

$$\boldsymbol{Min}\ \sum_{\boldsymbol{i=1}}^{\boldsymbol{m}}\ \sum_{\boldsymbol{j=1}}^{\boldsymbol{n}} \boldsymbol{x_{ij}}\, \boldsymbol{c_{ij}}. \tag{1}$$

The demand constraint is given by:

$$\sum_{\boldsymbol{j=1}}^{\boldsymbol{n}} \boldsymbol{x_{ij}} \leq \boldsymbol{a_{ij}},\ \ \boldsymbol{i} = \boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{m}, \tag{2}$$

while the supply constraint is given by:

$$\sum_{\boldsymbol{i=1}}^{\boldsymbol{m}} \boldsymbol{x_{ij}} \geq \boldsymbol{b_{ij}},\ \ \boldsymbol{j} = \boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{n}. \tag{3}$$

The non-negativity constraints are:

$$\boldsymbol{x_{ij}} \geq \boldsymbol{0},\ \ \boldsymbol{i} = \boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{m};\ \boldsymbol{j} = \boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{n}. \tag{4}$$

3.3 Degeneracy

In the study conducted by Rao & Mishra [25], the researchers proposed that a particular condition would emerge under specific circumstances in the context of their analysis. This condition occurs when the total number of filled cells in each matrix is less than the sum of the number of columns (m) and the number of rows (n), minus one (m+n–1). This finding has significant implications for the understanding of the problem being investigated. Further examination of this condition reveals that it can arise at certain points during the initial allocation process. Specifically, the condition may manifest when the first entry in the matrix simultaneously satisfies the requirements for both columns and rows. In other words, the initial allocation is such that it meets the criteria for both the column and the row in which it is placed, without violating any constraints. Additionally, the researchers identified another instance in which this condition can emerge. This occurs when the

values that are being subtracted and added within the matrix are equal. In this case, the balance between these values ensures that the condition is satisfied, and the matrix remains consistent with the theoretical framework outlined in [25]. A specific condition arises when the number of filled cells is less than the sum of the number of columns and rows minus one ($m+n-1$) [27]. This condition can exist during the initial allocation process when the first entry meets both column and row requirements, and/or when the subtracted and added values within the matrix are equal. These findings contribute to a deeper understanding of the problem at hand and provide valuable insights for further research in this field.

3.4 Basic Feasible Solution

A transportation problem is a well-studied optimization problem in operations research and management science. It deals with the distribution of goods or resources from a set of origins or sources (M) to a set of destinations or sinks (N), intending to minimize the total transportation cost. The problem is commonly formulated as a linear programming model, with a solution space that contains feasible allocations of goods between the sources and destinations. One of the key aspects of analyzing and solving the transportation problem is determining the basic feasible solutions. These solutions form the foundation for the optimization process, as they represent a subset of all possible allocations that satisfy the constraints and conditions of the problem. In this context, a solution to an $M \times N$ transportation problem is a basic feasible solution if the total number of allocations made between the sources and destinations is equal to the sum of the number of sources and the number of destinations minus 1 (i.e., $M+N-1$). This condition ensures that the basic feasible solution is non-degenerate, meaning that it has a unique solution for each variable in the system. It is important to identify basic feasible solutions because they play a crucial role in the optimization process, serving as the starting point for the search for the optimal solution. In many cases, basic feasible solutions can be found using well-established methods such as the northwest corner rule, the minimum cell cost method, or Vogel's approximation method. Once a basic feasible solution is obtained, iterative techniques such as the stepping stone method or the modified distribution method can be employed to refine the solution and eventually reach the optimal allocation.

3.5 Optimal Solution

In the context of transportation and logistics optimization, a basic feasible solution is considered optimal if it results in the lowest possible overall transportation cost. This is achieved by effectively allocating resources and determining the most efficient routes to transport goods from their origin to their destination points while satisfying supply and demand constraints. An optimal basic feasible solution is instrumental in maximizing cost savings, streamlining supply chain processes, and ultimately improving the overall performance of the transportation system. By carefully examining and comparing multiple feasible solutions, decision-makers can identify the most cost-effective strategy for their transportation needs, thereby significantly reducing expenses and enhancing the economic efficiency of their operations.

4. Results

Table 1 shows the distance of different destinations from the company's manufacturing unit. We denoted the destinations Gilgit with P_1, Chitral with P_2, AJK with P_3, and Swat with P_4.

Table 1
Distance of different places from the manufacturing unit

Destination	Gilgit	Chitral	AJK	Swat
Distance (km)	516	389	133	235

Table 2 shows the estimated transportation cost suggested by five different suppliers; i.e., S_1, S_2, S_3, S_4, and S_5.

Table 2
Transportation costs quoted by different suppliers to four different places

Suppliers	P_1	P_2	P_3	P_4
S_1	50	110	65	170
S_2	48	90	50	155
S_3	52	100	55	160
S_4	55	105	70	165
S_5	60	125	75	185

Table 3 shows the quantity offered to each supplier per month.

Table 3
The available quantity for each supplier

Suppliers	S_1	S_2	S_3	S_4	S_5
Quantity	7000	8200	5250	3900	5100

Table 4 shows the demand required by the four destinations per month.

Table 4
The demand of the destinations

Destination	P_1	P_2	P_3	P_4
Demand	4000	1000	9000	5000

By combining the values from Tables 1–4, the following model for deciding an optimal solution to cut the transportation cost of bakery products to different destinations is formed (Table 5), where D^* stands for demand and A^* denotes the availability, with $\sum a_i = 29{,}450$ and $\sum b_j = 19{,}000$, so $\sum a_i - \sum b_j = 10{,}450$.

Table 5
Demand and availability

Suppliers	P_1	P_2	P_3	P_4	A^*
S_1	50	110	65	170	7000
S_2	48	90	50	155	8200
S_3	52	100	55	160	5250
S_4	55	105	70	165	3900
S_5	60	125	75	185	5100
D^*	4000	1000	9000	5000	–

We have added a dummy place P_5 with the requirement of 10,450 units and zero transportation cost, as displayed next for the balanced transportation problem in Table 6.

Table 6
Addition of one destination for balancing

Suppliers	P_1	P_2	P_3	P_4	P_5	A^*
S_1	50	110	65	170	0	7000
S_2	48	90	50	155	0	8200
S_3	52	100	55	160	0	5250
S_4	55	105	70	165	0	3900
S_5	60	125	75	185	0	5100
D^*	4000	1000	9000	5000	10,450	–

In this study, we have meticulously implemented five distinct methodologies to obtain the preliminary basic feasible solutions, each catering to diverse problem domains and optimization criteria. These methods are strategically designed to provide a comprehensive understanding of the solution space, thereby contributing to the robustness and reliability of the overall research outcomes. The methods are:

i. Northwest corner;
ii. Least cost;
iii. Vogel approximation;
iv. Row minima;
v. Column minima.

4.1 Northwest Corner Method

The following steps are involved in this method:

Step 1 – *Initialize the first cell*: Begin by allocating the first cell in the top-left corner of the transportation matrix, following the northwest corner rule. Assign the value x_{11} to the minimum of a_1 and b_1 ($x_{11} = min(a_1, b_1)$) [28,29], where a_1 represents the supply of the first row and b_1 represents the demand of the first column.

Step 2 – *Update rows and columns*: Determine if x_{11} equals a_1 or b_1. If x_{11} is equal to a_1, it means that the first row's supply has been satisfied. In this case, delete row 1 and update the demand of the first column by subtracting a_1 from b_1 ($b_1 = b_1 - a_1$). If x_{11} equals b_1, it signifies that the demand of the first column has been met. Consequently, cross out column 1 and update the supply of the first row by subtracting b_1 from a_1 ($a_1 = a_1 - b_1$).

Step 3 – *Handle degenerate cases*: If a_1 is equal to b_1 ($x_{11} = a_1 = b_1$), it implies that both the supply of the first row and the demand of the first column have been satisfied. As a result, delete both row 1 and column 1. This situation indicates a degenerate initial basic feasible solution, meaning at least one basic variable will be zero. Thus, there will be fewer than $m + n - 1$ positive basic variables in the basic feasible solution.

Step 4 – *Repeat the process*: Continue to iteratively perform Steps 1–3 until all supply and demand values have been exhausted. This process can only be completed if the problem is balanced, meaning that the total supply equals the total demand.

By following these detailed steps, the northwest corner rule algorithm can be effectively employed in a research paper to determine an initial feasible solution for transportation problems.

The Northwest corner method is a straightforward and efficient technique used to solve transportation problems. Below, we provide a comprehensive and detailed step-by-step guide for its implementation:

i. *Balancing the problem*: Start by examining if the given transportation problem is balanced or unbalanced. A problem is considered balanced if the total supply equals the total demand; otherwise, it is unbalanced. If necessary, balance the problem by adding a dummy row or column to account for the difference between supply and demand;
ii. *Initializing allocation*: Begin at the top-left corner cell of the transportation table. Allocate the maximum number of units possible, which is equal to the minimum value between the available supply and demand in the current cell;
iii. *Adjusting supply and demand*: Update the supply and demand values in the corresponding rows and columns by subtracting the allocated units from the initial supply and demand figures;
iv. *Moving horizontally*: If the demand for the current cell has been satisfied, move to the next cell in the same row and the second column;
v. *Moving vertically*: If the supply for the current cell has been satisfied, move to the next cell in the same column and the second row;
vi. *Deleting columns*: Once the demand in a column has been met, remove the column from the transportation table;
vii. *Deleting rows*: Once the supply in a row has been met, remove the row from the transportation table;
viii. *Iterative process*: Continue the allocation process, moving horizontally or vertically as required and deleting rows or columns as needed, until all the supply and demand quantities are satisfied.

The obtained initial feasible solution was: $X_{11} = 4000$, $X_{12} = 1000$, $X_{13} = 2000$, $X_{23} = 7000$, $X_{24} = 1200$, $X_{34} = 3800$, $X_{35} = 1450$, $X_{45} = 3900$, and $X_{55} = 5100$, with a total transportation cost of 1,588,000.

4.2 Least Cost Method

This technique works by applying operation on the least cost value of the full transportation table. The allocation order is initiated from the lowest cost to the last [30,31].

The following steps are involved in this method:

Step 1 **–** Select the cell, which has the least cost value c_{ij} and then assign maximum value to the cell by taking $x_{ij} = \min(a_i, b_j)$.

Step 2 – If $x_{ij} = a_i$, the *i*-th row should be erased. Otherwise, if $x_{ij} = b_j$, then the *j*-th column must be deleted.

Step 3 – Repeat Steps 1-–2 with a reduced matrix till all the supply and demand are distributed.

The obtained initial feasible solution was: $X_{15} = 7000$, $X_{21} = 4000$, $X_{23} = 750$, $X_{25} = 3450$, $X_{33} = 5250$, $X_{42} = 900$, $X_{43} = 3000$, $X_{52} = 100$, and $X_{54} = 5000$, with a total transportation cost of 1,760,250.

4.3 Vogel's Approximation Method

Vogel's approximation method is a step-by-step process utilized to compute an initial basic feasible solution for transportation problems. This method is preferred over other methods examined in the paper as the initial basic feasible solution derived from it is either the optimal solution or near optimal. Although somewhat more intricate than other methods, it incorporates the concept of penalty cost [32,33]. The comprehensive steps for executing VAM are provided below:

Step 1 – Determine the lowest cost c_{ij} and the next lowest cost c_{ik} for each row. Subsequently, calculate the penalty cost by finding the difference between c_{ik} and c_{ij}. Display these values alongside the transportation tableau. Continue to compute the variances for each column.

Step 2 – Identify the largest penalty cost, which is the greatest difference among the rows or columns. In the event of a tie, select any one of them randomly. Let the largest penalty cost be in the *i*-th row and let c_{ij} represent the smallest cost in that row. Allocate $x_{ij} = \min(a_i, b_j)$, and either eliminate or cross out the *i*-th row or the *j*-th column.

Step 3 – Compute the new penalty cost for each row or column of the reduced matrix. Repeat Steps 1–2 until all demand and supply requirements are met.

The obtained initial feasible solution was: $X_{11} = 4000$, $X_{12} = 1000$, $X_{13} = 2000$, $X_{23} = 7000$, $X_{24} = 1200$, $X_{34} = 3800$, $X_{35} = 1450$, $X_{45} = 3900$, and $X_{55} = 5100$, with a total transportation cost of 1,584,000.

4.4 Row Minima Method

The row minima method for solving transportation problems is a simple and effective approach that begins with the smallest cost cell c_{ij} in the first row of the transportation matrix [34]. This method allocates $x_{1j} = \min(a_1, b_j)$, where a_1 represents the supply at the origin and b_j represents the demand at the destination. It has three primary cases that dictate its allocation process. These cases are described in detail below:

i. If the demand of the destination is satisfied ($x_{1j} = b_j$), the first column is crossed out, and the supply at the origin a_1 is updated to reflect the allocation by setting $a_1 = a_1 - b_j$;
ii. If the supply from the origin is exhausted ($x_{1j} = a_1$), the first row is removed from the matrix, and the demand at the destination b_j in the *j*-th column is $b_j = b_j - a_1$;
iii. If both the demands from the destinations and supplies from the origins are completely satisfied ($x_{1j} = a_1 = b_j$), the first row and the *j*-th column are both removed from the matrix.

The obtained initial feasible solution was: $X_{15} = 7000$, $X_{21} = 4000$, $X_{23} = 750$, $X_{25} = 3450$, $X_{33} = 5250$, $X_{42} = 900$, $X_{43} = 3000$, $X_{52} = 100$, and $X_{54} = 5000$, with a total transportation cost of 1,760,250.

4.5 Column Minima Method

The column minima method is an efficient technique for solving transportation problems, which focuses on the column of the transportation problem [35]. The method initiates with the least-cost cell c_{ij}, in the first column and assigns $x_{i1} = \min(a_i, b_1)$. Although it does not involve many steps, it presents some cases that need to be addressed. These cases are elaborated below in a comprehensive manner.

i. *Demand satisfaction at the destination*: The demand of the destination is satisfied, meaning that $x_{i1} = b_1$. Consequently, the first column is eliminated, and the value of a_i is updated to $a_i - b_1$. This signifies that the supply from the origin is reduced by the amount of demand satisfied at the destination.
ii. *Exhausted supply from the origin*: In this case, the *i*-th row is removed, and the demand for b_1 in the first column is adjusted to $b_1 - a_i$. This adjustment reflects the reduced demand at the destination due to the exhausted supply from the origin.
iii. *Complete satisfaction of demands and supplies*: This case requires the deletion of both the *i*-th row and the first column, indicating that the supply and demand have been met entirely, and further calculations within this row and column are no longer necessary.

The obtained initial feasible solution was: $X_{13} = 550$, $X_{14} = 1100$, $X_{15} = 5350$, $X_{21} = 4000$, $X_{22} = 1000$, $X_{23} = 3200$, $X_{33} = 5250$, $X_{44} = 3900$, and $X_{55} = 5100$, with a total transportation cost of 1,597,000.

The total transport cost by Vogel's approximation method is less than the other four methods. The result comparison in graphical form is presented in Figure 1.

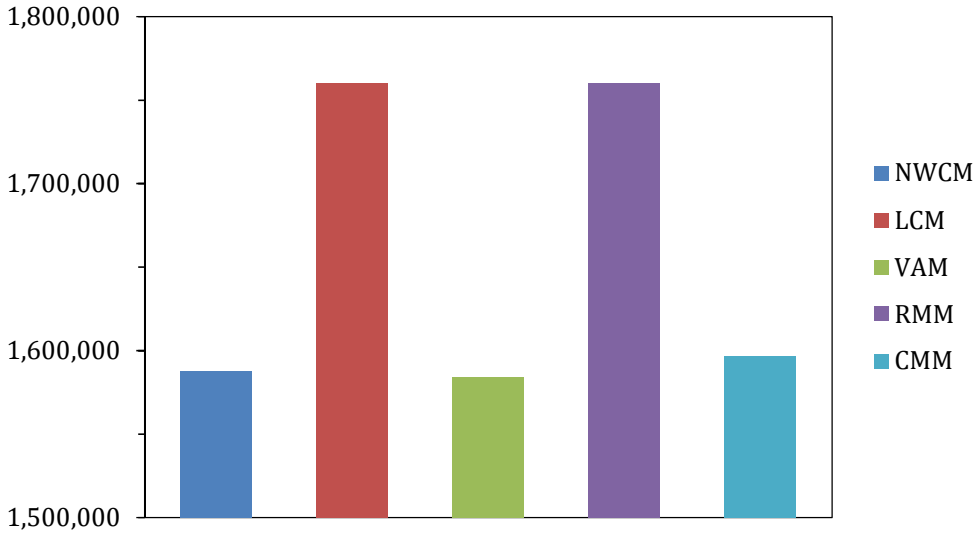

Fig. 1. Result comparison of all the approximation methods

This work performs a thorough investigation of the solution space by using these five approaches to find viable solutions. The outcomes of each technique might be useful starting points for other optimization procedures like transportation algorithms or sophisticated heuristics. Comparing the results of several approaches can also reveal the advantages and disadvantages of each strategy regarding certain problem areas and optimization criteria. It is important to recognize that the early answers found using these techniques are only approximations at this point and might not accurately represent the overall ideal solution. The efficiency of the selected approach depends on the traits, restrictions, and goals of the challenge, as it does with every optimization problem. As a result, it is advised to do more study and use optimization techniques to enhance and improve the discovered solutions for use in actual applications in real-world circumstances. In conclusion, the careful use of these five unique approaches enhances the robustness and trustworthiness of the study findings by fostering a better comprehension of the transportation problem solution space. The knowledge gathered from this study can pave the way for more sophisticated optimization techniques and help

decision-makers in many sectors make well-informed decisions for effective resource allocation and supply chain management.

5. Conclusions

Our research has identified an optimal solution for dairy product suppliers to minimize both transportation and fuel costs. By comparing various approximation methods, we have determined that the Vogel approximation method is the most effective for this specific food chain in Pakistan. The comparative analysis, as depicted in the graph, underscores the superiority of the Vogel method in terms of cost savings among the alternatives. By implementing this best practice, dairy product suppliers can significantly reduce their operational expenses, thereby benefiting both their businesses and their customers.

Ultimately, this research contributes to the development of efficient and sustainable supply chain management practices in the dairy industry. Our findings can serve as a valuable resource for businesses seeking to optimize their operations and improve their overall performance in the competitive marketplace.

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Conflicts of Interest

The author declares no conflicts of interest.

References

[1] Christopher, M. (2016). *Logistics and Supply Chain Management* (5th ed.). Pearson.
[2] Coyle, J. J., Langley, C. J., Novack, R. A., & Gibson, B. (2016). *Supply chain management: A logistics perspective* (10th ed.). Cengage Learning.
[3] Langley, C. J. (2016). *The state of logistics outsourcing: 2016 Third-Party Logistics Study*. Capgemini Consulting, Penn State University, and Penske Logistics.
[4] Vidal, C. J., & Goetschalckx, M. (1997). Strategic production-distribution models: A critical review with emphasis on global supply chain models. *European Journal of Operational Research, 98*(1), 1-18. https://doi.org/10.1016/S0377-2217(96)00397-2.
[5] Ahuja, R. K., Magnanti, T. L., & Orlin, J. B. (2014). *Network flows: Theory, algorithms, and applications*. Prentice Hall.
[6] Waters, D. (2010). *Global logistics: New directions in supply chain management* (6th ed.). Kogan Page.
[7] Notteboom, T., & Rodrigue, J. P. (2009). The future of containerization: Perspectives from maritime and inland freight distribution. *GeoJournal, 74*(1), 7-22. https://doi.org/10.1007/s10708-008-9187-9.
[8] Stank, T. P., Goldsby, T. J., Vickery, S. K., & Savitskie, K. (2015). Logistics service providers: An empirical examination of the size-efficiency relationship. *Journal of Business Logistics, 25*(2), 179-201. https://doi.org/10.1002/j.2158-1592.2004.tb00177.x.
[9] Aven, T. (2016). Risk assessment and risk management: Review of recent advances on their foundation. European *Journal of Operational Research, 253*(1), 1-13. https://doi.org/10.1016/j.ejor.2015.12.023.
[10] Seuring, S., & Müller, M. (2008). From a literature review to a conceptual framework for sustainable supply chain management. *Journal of Cleaner Production, 16*(15), 1699-1710. https://doi.org/j.jclepro.2008.04.020.
[11] Crainic, T. G., & Laporte, G. (2013). Planning models for freight transportation. *European Journal of Operational Research, 97*(3), 409-438. https://doi.org/10.1016/S0377-2217(97)00177-1.
[12] Nagurney, A. (2010). Supply chain network design: Optimization models, methodologies, and applications. *International Journal of Production Economics, 128*(1), 1-12. https://doi.org/10.1016/j.ijpe.2010.07.025.
[13] Mentzer, J. T., Min, S., & Bobbitt, L. M. (2001). Toward a unified theory of logistics. *International Journal of Physical Distribution & Logistics Management, 31*(2), 99-121. https://doi.org/10.1108/09600030110385815.
[14] Rushton, A., Oxley, J., & Croucher, P. (2014). *The handbook of global supply chain management*. Sage Publications.
[15] Halldórsson, Á., & Kovács, G. (2010). The sustainable agenda and energy efficiency: Logistics solutions and supply chains in times of climate change. *International Journal of Physical Distribution & Logistics Management, 40*(1/2), 5-13. https://doi.org/10.1108/09600031011020432.

[16] Abioye, O. F., Dulebenets, M. A. M., Kavoosi, Pasha, J., & Theophilus, O. (2019). Vessel Schedule Recovery in Liner Shipping: Modeling Alternative Recovery Options. *IEEE Transactions on Intelligent Transportation Systems, 22*(10), 6420-6434. https://doi.org/10.1109/TITS.2020.2992120.

[17] Dulebenets, M. A. (2019). Minimizing the total liner shipping route service costs via application of an efficient collaborative agreement. *IEEE Transactions on Intelligent Transportation Systems, 20*(1), 123-136. https://doi.org/10.1109/TITS.2018.2794998.

[18] Chinedum, O. (2018). Port congestion determinants and impacts on logistics and supply chain network of five African ports. *Journal of Sustainable Development of Transport and Logistics, 3*, 70-82. https://doi.org/10.14254/jsdtl.2018.3-1.7.

[19] Gumuskaya, V., van Jaarsveld, W., Dijkman, R., Grefen, P., & Veenstra, A. (2020). A framework for modelling and analysing coordination challenges in hinterland transport systems. *Maritime Economics & Logistics, 22*, 124-145. https://doi.org/10.1057/s41278-019-00139-1.

[20] Loch, G. V. & Silva, A. C. L. D. (2014). A Computational Study on the Number of Iterations to Solve the Transportation Problem. *Applied Mathematical Sciences, 8*, 4579-4583. https://doi.org/10.12988/ams.2014.48958.

[21] Kumaraguru, S., Kumar, S. B., & Revathy, M. (2014). Comparative Study of Various Methods for Solving Transportation Problem. *International Journal of Scientific Research, 3*(9), 244-246. https://doi.org/10.15373/22778179/september2014/81.

[22] Verma, V., & Puri, M. C. (2022). On a paradox in linear fractional transportation problems. In *Recent developments in mathematical programming* (pp. 413-424). CRC Press.

[23] Afroz, S. & Hassan, M. B. (2015). A Computer Oriented Method for Solving Transportation Problem. *Dhaka University Journal Science, 63*(1), 1-7. https://doi.org/10.3329/dujs.v63i1.28216.

[24] Bera, R.K., Mondal, S.K. (2020). Analyzing a Two-Staged Multi-objective Transportation Problem Under Quantity Dependent Credit Period Policy Using q-fuzzy Number. International Journal of Applied and Computational Mathematics, **6**, 146. https://doi.org/10.1007/s40819-020-00901-7

[25] Rao, K. C. & Mishra, S. L. (2005). *Operations Research*. Alpha Science International.

[26] Sen, S., Sahoo, L., Tiwary, K., Simic, V., & Senapati, T. (2023). Wireless Sensor Network Lifetime Extension via K-Medoids and MCDM Techniques in Uncertain Environment. Applied Sciences, 13(5), 3196. https://doi.org/10.3390/app13053196.

[27] Kacher, Y., & Singh, P. A. (2021). Comprehensive Literature Review on Transportation Problems. *International Journal of Applied and Computational Mathematics, 7*, 206. https://doi.org/10.1007/s40819-021-01134-y.

[28] Taha, H. A. (2007). *Operations Research: An Introduction* (8th ed.). University of Arkansas, Fayetteville.

[29] Panneerselvam, R. (2006). *Operations Research* (2nd ed.). Prentice-Hall of India Private Limited.

[30] Ajibade, A. D. & Babarinde, S. N. (2013). On the Use of Transportation Techniques to Determine the Cost of Transporting Commodity. *International Organization of Scientific Research Journal of Mathematics, 6*(4), 23-28. https://doi.org/10.9790/5728-0642328.

[31] Gupta, P. K. & Hira, D. S. (2005). *Operations Research*. Chand & Company Ltd, New Delhi.

[32] Liu, C. H. & Trung, L. L. (2013). The Study of Using Linear Programming to Solve Flower Transport Problem. *Journal of Convergence Information Technology, 8*(8), 1116-1124. https://doi.org/10.4156/jcit.vol8.issue8.133.

[33] Ali, U. A. & Mustafa, A. M. (2013). Comparisons of Transportation Algorithms Using Data from Katsina State Transport Authority, Katsina State, Nigeria. *Nigerian Journal of Basic and Applied Science, 21*(3), 201-214. https://doi.org/10.4314/njbas.v21i3.17.

[34] Srinivasan, G. (2010). *Operations Research Principles and Applications* (2nd ed.). PHI Learning Private Limited, New Delhi.

[35] Rodrigue, J. P. (2020). *The Geography of Transport Systems* (3rd ed.). Routledge.